Modelling a Sub-10 Second 100m Sprinter Using Newton’s Equations of Motion

By Jeremy Richmond

ABSTRACT

After the exploits of Usain Bolt in 2008 and 2009, many have asked: how it is possible that a human could run so fast? For rivals seeking to close the gap, strength training, the central element of the modern training paradigm for sprinters, offers only limited proven benefits for increasing maximal running velocity, which is the apparent key to Bolt’s domination. This paper discusses alternate training strategies. A model of forces generated in a sub-10 sec 100m was created based on Newton’s equations of motions and data from world-class sprint performances. It shows that after 30m, force in the horizontal direction is quite small, little more than body weight. At such a low level, the influence of maximum strength diminishes and the rate of force development becomes the predominant factor. Improvement of a sprinter’s maximal velocity requires more force production within the same ground contact times. This calls for greater training specificity, with more emphasis on increasing movement velocity and less on force production. The author suggests that better results may be achieved through explosive strength training such as plyometric exercises with a horizontal emphasis carried out with less force, reduced ground contact time and greater joint velocity.

AUTHOR

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Introduction

After the exploits of Usain Bolt (JAM) at the 2008 Olympic Games in Beijing, many in the audience will have asked: how it is possible that a human could run so fast? Four seconds (34m) into the final of the 100m, the other competitors seemed to be a match for Bolt, which would suggest they possess similar starting and acceleration qualities. This changed shortly thereafter as Bolt’s superior top-end velocity became evident to all. At 6.2 sec into the race the distance between him and silver medallist Richard Thompson (TRI) was 0.7m but barely a second later (at 7.3 sec), when Bolt was 73.3m down the track, the difference had lengthened to 1.2m.

It appeared that for the first third of the distance there was little difference between the finalists, but from that point onwards Bolt clearly distinguished himself as the “fastest man on earth” by pulling away from the field and setting a world record of 9.69 sec (see Figure 1).
The fact that the above training methods provide little significant benefit to sprint performance will disappoint those chasing Bolt and trying to bridge the large gap that exists. But perhaps their exercise methods need more specificity in order to provide better transfer of strength gains to sprinting. More specificity of movement in training makes sense because even when different exercises involve identical muscle groups the specific movement pattern used in training is where most of the strength improvement occurs. Therefore, useful improvements in strength for sprinters must be initiated in the muscles that generate the forces with the same pattern observed during sprinting. Moreover, we know that the greatest strength gains will occur at or near the training velocity. It seems that only through training with the same pattern and velocity will the coordination of agonist and synergist muscles improve effectively.

Therefore, what must be addressed in the design of training programmes for elite sprinters is whether and how well the forces in training mimic the requirements of sprinting.

Presumably, they have been auditing their training programmes in search of a shortfall that can be improved on. One aspect that they may be looking at is their strength training programme, as many previous sprint champions have demonstrated significant strength. However, the use of heavy resistance training, the means most accepted by modern coaches for increasing both strength and sprinting performance, has been demonstrated to have little effect on running velocity. One study has shown an improvement when heavy resistance is combined with sprint training, but we can’t be certain that the improvement was not due to the sprint training exclusively. Moreover, little significance has been proven for a number of other strength-oriented training methods such as Olympic lifting, power lifting, ballistics, plyometric training and resisted running, especially when they are compared to sprint training itself.

Some researchers believe that greater top-end running velocity is achieved with greater vertical ground forces. Others have shown that horizontal thrust is necessary for for-
ward movement and that maximal velocity is more dependent on horizontal forces than vertical forces. It would be great if we could examine these forces throughout the race, especially at the world-class level, to allay any doubts between the theories. But for now we can say that sprinting is characterised by short force production times. In a study of the relationship between strength measures and sprinting performance, the best predictor of 2.5m starting performance was found to be the peak force generated during the concentric contraction of a jump (r=0.86) from a similar biomechanical position to that held in the starting blocks. The same study also found that the concentric force applied at 100 ms from the start of a loaded jumping action correlated with maximum sprinting speed (r=0.80). In addition, strong correlations were found with the countermovement jump (r=-0.79) and maximum force during a jump take-off (r=-0.79) and maximal running velocity. These results show that sprint performance is related to the rate of force production and may also be related to explosive movement.

The velocity of a runner at full speed relates directly to the backswing velocity of his/her leg. Very strong correlations have been observed between running velocity in male participants and the peak angular thigh pushing velocity (r=0.98) whilst peak angular velocity of the lower leg (r=0.96) was also found to correlate extremely well. Similarly in females, peak angular velocity of the lower leg was found to be a strong predictor of sprinting velocity (r=0.98). These results show that sprint performance correlates very strongly with velocity of movement in the propulsive limbs, at least when running at high velocities.

The question is whether improving force generation or velocity of movement is going to be enough to catch Bolt. In order to address this, we need first to understand the mechanisms by which Bolt may have an advantage over his competitors. Although a large amount of biomechanical research on the sprints has been carried out the context of IAAF projects at the Olympic Games and World Championships in Athletics, there remains a dearth of information that allows us to understand the mechanisms related to how world-class sprinters run faster than everyone else. Therefore, we have to use modelling to give us an insight into what might transpire in a world record sprint race. Such an insight might reveal some clues as to where improvements can be gained.

This paper, therefore, intends to approximate the mechanisms that propel world-class sprinters to their speeds by using Newton’s equations of motion.

**Newtonian Modelling**

**Force and Velocity**

To move forward, any runner must produce horizontal force against the ground. The sprinter must produce vertical force but this need only be enough to allow him/her to reposition the legs for the following step and horizontal force generation. If the sprinter chooses to generate more vertical force this may allow more time in which to create backward movement of the repositioned leg in order to generate horizontal force. In the absence of data on force production for world-class sprinters, we will approximate the forces using equations of physics.

In 1686 Sir Isaac Newton (1642-1727) stated three natural laws relating force and motion. These laws provide us with equations of motion that relate force production with ensuing velocity. The equation of impulse governs the velocity:

$$Ft = m(u-v)$$

where $u-v$ represents the change in velocity ($u$ is initial velocity and $v$ is the final velocity), $t$ is the change in time between each step and $m$ is the mass of the sprinter. The impulse that results in the forward motion of the sprinter is only applied when the foot of the sprinter is in contact periodically with the ground. Therefore, the equation for impulse is:

$$F_{total} \times t_{ground \ contact \ time} = m(u-v)$$
**Retarding Forces Due to Friction**

Whilst it can be seen that horizontal force is directly related to horizontal velocity, we must bear in mind that the sprinter might be producing greater forces that are dissipated as friction, such as that due to air resistance. The sprinter is slowed down by force due to air resistance (drag), which is proportional to the cross sectional area of the sprinter and the square of the velocity that the sprinter is running. The equation of the force due to air resistance from previous published literature\textsuperscript{31,32} is:

\[ F_{\text{drag}} = 0.549v^2A \] (in Newtons)

where \( v \) is the velocity and \( A \) is the frontal area of the runner. The area \( A \) is shown to be approximately 0.5m\(^2\) for a sprinter\textsuperscript{33}. Air resistance slows down the sprinter during the flight phase and must also be overcome during the stance phase. In addition, top quality sprinters experience a loss in horizontal velocity of 2-3\% during ground contact with this decrease being 5-6\% in sprinters of lesser quality\textsuperscript{34}. Without exact data of this loss during the contact phase for our model, we will estimate that a loss of 3\% occurs. Therefore, this has to be taken into account as an increased demand for force production in this model in order to maintain the calculated velocity after each step. With this in mind the complete equation for force becomes:

\[ F_{\text{total}} = F_{\text{ideal}} + F_{\text{drag}} + F_{\text{ground contact losses}} \]

<table>
<thead>
<tr>
<th>Distance</th>
<th>Time averaged for two sprinters in each 10m section of the 1988 Olympic 100m [sec]</th>
<th>Time for Maurice Greene in each 10m section of a 9.86 sec 100m [sec]</th>
<th>Differential [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10m</td>
<td>1.74</td>
<td>1.71</td>
<td>1.75</td>
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<tr>
<td>20m</td>
<td>2.76</td>
<td>2.75</td>
<td>0.36</td>
</tr>
<tr>
<td>30m</td>
<td>3.70</td>
<td>3.67</td>
<td>0.82</td>
</tr>
<tr>
<td>40m</td>
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</tr>
<tr>
<td>50m</td>
<td>5.43</td>
<td>5.42</td>
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<tr>
<td>60m</td>
<td>6.29</td>
<td>6.27</td>
<td>0.32</td>
</tr>
<tr>
<td>Avg. difference</td>
<td></td>
<td></td>
<td>0.65</td>
</tr>
</tbody>
</table>

**Method**

To develop a model of a sub-10 sec 100m sprinter, data from numerous papers were combined\textsuperscript{35,36,37}. When reaction time is neglected, the average times of the sprinters in each study compare well, such that the greatest differential is 1.75\% over the first 10m with the average difference in times at each interval up to 60m being 0.65\% (see Table 1). From this we equated the average number of steps taken and the average ground contact time with that of the instantaneous velocity (Table 2).

Both studies used video analysis for each 10m interval. In addition, laser guns (LAVEG Sport, Germany) were used to determine the instantaneous velocity of former world record holder Maurice Greene (USA) on his way to a 9.86 sec 100m\textsuperscript{37}. The methods of analysis were justified in each paper. Lastly, the mass of Greene was quoted at 75kg\textsuperscript{35}.

It can be seen from the stride model of a sub-10 sec sprinter (Figure 2) that horizontal force production diminishes rapidly from the start of the race until about 30m, at which point it continues to diminish but at a markedly slower rate. Likewise it can be seen that ground contact time behaves in a similar fashion with both relationships tending towards a plateau. If Bolt’s competitors fall behind from 30m onwards it seems that they only need to produce more horizontal force or increase ground
Table 2: Number of steps and averaged ground contact times\textsuperscript{36} for male Olympic 100m Finalists equated with instantaneous velocity of Maurice Greene in 9.83 sec 100m sprint (data from BRÜGGEMAN & GLAD\textsuperscript{36} and KERSTING\textsuperscript{37})

<table>
<thead>
<tr>
<th>Distance</th>
<th>Averaged steps per 10m section for sprinters in the 1988 Olympic 100m</th>
<th>Average ground contact time for the last step in each 10m section</th>
<th>Instantaneous velocity of Maurice Greene in each 10m section of a 9.86 sec 100m</th>
</tr>
</thead>
<tbody>
<tr>
<td>10m</td>
<td>7 [n] 124.5 [m/sec]</td>
<td>8.71 [m/sec]</td>
<td>8.71 [m/sec]</td>
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<tr>
<td>20m</td>
<td>5 [n] 95.5 [m/sec]</td>
<td>10.47 [m/sec]</td>
<td>10.47 [m/sec]</td>
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<tr>
<td>30m</td>
<td>5 [n] 86.0 [m/sec]</td>
<td>11.14 [m/sec]</td>
<td>11.14 [m/sec]</td>
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<tr>
<td>40m</td>
<td>4 [n] 83.75* [m/sec]</td>
<td>11.50 [m/sec]</td>
<td>11.50 [m/sec]</td>
</tr>
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<td>50m</td>
<td>4 [n] 81.5 [m/sec]</td>
<td>11.67 [m/sec]</td>
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</tr>
<tr>
<td>60m</td>
<td>4 [n] 81.0 [m/sec]</td>
<td>11.80 [m/sec]</td>
<td>11.80 [m/sec]</td>
</tr>
</tbody>
</table>

\*Data smoothed from 86 to 83.75 (i.e. (86 - 81.5)/2)

Results and Discussion

Figure 2: Stride model for a sub-10 sec 100m sprinter: comparison of horizontal force approximated per step per 10m interval and ground contact time at the end of each 10m interval for the first 60m

Contact time in order to increase their velocity according to Newton’s equation of motion; $Ft = m(u-v)$. However, we may be limited in ground contact time by the velocity at which they are running. If the legs are to be considered as being of fixed length, then the faster a sprinter runs must mean that less time is available to be in contact with the ground. Therefore it would seem that the only mechanism by which sprinters can influence their speed is through increased force production within the ever-diminishing time in which they are in contact with the ground. How the sprinter should achieve this is a matter for debate.

As mentioned above, it is generally accepted by coaches that sprinting speed can be improved by means of strength training\textsuperscript{38}. The main methods employed include traditional strength training or explosive strength training. What is of interest is how these methods affect force and force production time, which is referred to as ground contact time in sprinting. Studies show that as a result of a traditional strength training programme, the time to produce 3000N (30% of maximum) reduced by 31\%\textsuperscript{39}. Similarly, explosive type strength training resulted in a reduction of 34% in the time to produce 3000N\textsuperscript{40}. However, the time to produce 500N did not change as a result of strength training but this time was reduced by 18% through explosive training (Figure 3 and Figure 4). This is of interest to sprinters and coaches as the forces are of relevant magnitudes to those experienced in sprinting.

Our calculation of forces produced in a sub-10 sec 100m reveals that horizontal force production is quite low, approximately 20kg (~200N) around the 30m point. However, for
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An alternate and perhaps more relevant way of looking at this data is that within the time frame of 50 ms or at 100 ms the amount of force produced is enhanced from explosive strength training to a greater extent than from traditional strength training.

In order to provide the greatest transfer of strength gains into sprinting it is suggested that the training exercise used mimic sprinting in terms of specificity of movement and velocity15,16,17,18.

With regards to Newton’s equation of motion for impulse, we need to consider force production and force production time (ground contact time). Plyometric training is generally classified as explosive training and is regarded as sprint specific15,41. This type of training involving substantial horizontal exercise has consistently proven to reduce sprint times over 10m 4,5,43.

This brings into question the reasons why this type of training method has not proven much beyond 10m4,5. Research has shown that resultant forces produced during plyometric exercise, such as maximal hopping, at speeds above 9.96 m/sec. Researchers16,41 report a net average vertical propulsion force of 615N or 62.7kg at 9.59 m/sec and of 621N or 63.3kg at supramaximal towing speeds of 10.82 m/sec where horizontal force is aided. Assuming therefore that the vertical force is around 81.2kg this would equate to a net resultant force of around 824N or 84kg somewhere near the 30m point. If the sprinter is 75kg the total vertical force would then be 1358N, which is much less than that produced in a counter-movement jump of around 2879N42. For the sake of argument we could surmise that the total resultant force around the 30m point, where Bolt separated himself from his competitors in Beijing, is close to 1400N (143kg). In reference to the charts comparing traditional strength training with explosive training (Figure 3, Figure 4) we can see that the time to produce 1400N through explosive strength training reduces whereas there is negligible change from a traditional strength training programme.

Figure 3: Force production time after 24 weeks of strength training (data from HAKKINEN, KOMI & ALEN43)

Figure 4: Force production time after 24 weeks of explosive strength training (data from HAKKINEN, KOMI & ALEN43)

the athlete to know what force to produce in training the horizontal force needs to be combined with vertical forces, for which the highest measured propulsive amount is 797N or 81.2kg at 9.96 m/sec41. We cannot speculate as to whether the vertical force is higher or lower.
are around two times greater than in maximal running\textsuperscript{18}. Equally of interest is the force production time, which is reported to be 1.9 times longer in maximal hopping compared to sprinting whilst the speed of execution of the exercise is slower. It would seem plausible to utilise maximal hopping type exercise with a lower magnitude of force production - but still higher than sprinting – quicker movement and shorter ground production times to have a greater specificity to sprinting.

**Conclusion**

The model of a sub-10 sec 100m sprinter shows that the forces in the horizontal direction are quite small and resultant forces are little more than body weight at speed. At such low force levels, the influence of maximal strength diminishes and the rate of force development becomes the predominant factor\textsuperscript{20}. Improvements to speed require more force production within the same ground contact time and may be better achieved through explosive strength training. Perhaps greater improvements can be achieved by modifying explosive strength training to accommodate more specific force production times to that in sprinting. It is suggested that explosive type training, such as plyometric exercises with more horizontal emphasis be carried out with less force, reduced ground contact time and greater joint velocity, be tried.

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